

Optimal control of a planar robot manipulator based on the Linear Quadratic Inverse-Dynamics design

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Abstract. To ensure the correct positioning of the end-effector of robot manipulators is one of the most important objectives of the robotic systems control. Lack of reliability in track the reference trajectory, as well as in the desired final positioning compromises the quality of the task to be performed, even causing accidents. The purpose of this work was to propose an optimal controller with an inner loop based on the dynamic model of the manipulator and a feedback loop based on the Linear Quadratic Regulator, in order to ensure that the end effector is in the right place, at the right time. The controller was compared to the conventional PID, presenting better performance, both in the transient response, eliminating overshoot, and steady state, eliminating stationary error.

Keywords: Linear Quadratic Regulator. Computed-torque control. Feedback control.

1 Introduction

Robot manipulators are programmable machines capable of performing a wide variety of tasks, such as: welding, pick up and put, parts assembling, package and paint spraying. To accomplish the tasks, there are a dependency of the motions of the end-effector or the interaction forces between the end-effector and the environment where the robot is working (LAMMERTS, 1993). A planar manipulator is a type of manipulator where all the links move in parallel planes to one another.

Before the robot accomplish any work, it is necessary to position the end-effector in the right place at the right moment. To do this, it is necessary to determine the motion of each link. As the motion is produced by motors that drive the corresponding joint, it is necessary to solve the tracking control problem, that is, to determine the actuator torques and forces that make the manipulator follow the desired trajectory as well as possible, in order to ensure that the robot realizes the task with the desired performance.

Using the desired end-effector trajectory and the inverse dynamics model of the manipulator, it is possible to calculate the torques necessities to perform the task (MOOLAM, 2013). However, although model equati-

ons are well known, only imprecise knowledge of the parameters is available. This so-called parametric uncertainty may be caused by an unknown load at the end-effector, poorly known inertias, uncertain and slowly time-varying friction parameters. Thus, to realize the desired tasks with the necessary performance, a controller with feedback loop is required (LAMMERTS, 1993).

The purpose of this work was to study a control system with an inner feedforward loop based on the inverse dynamics of the manipulator and an outer linear feedback loop based on the Linear Quadratic Optimal Control. The controllers obtained was compared with the traditional PID controllers. This work also evaluated the effect of the change of the weights of the matrix Q on the kinematic and dynamics variables of the manipulator.

Three hypotheses can be formulated about the proposed controller: (1) This controller provide better tracking position than the conventional PID; (2) less effort (torque) is required from the actuator to perform the desired motion and (3) the tuning of the proposed controller is easier than the conventional PID.

The importance of the study is justified by the fact that it is necessary to find controls that provide robust-

ness and optimized performance for robot manipulators.

2 Related Works

Since many tasks performed by manipulator robots depend on the location in the space of the end-effector, considerable effort has been made to create position controllers to ensure the correct positioning of the manipulated tool. In this section, a brief review about some related works is shown.

Khairudin, Mohamed & Husain (2011) applied the Linear Quadratic Regulator to control the position of the end-effector of a flexible link robot manipulator. The results obtained with the controller utilized were compared with the ones obtained utilizing a conventional PID controller. Both techniques were able to reach the desired angular positions however the LQR control exhibited a reduction of 32.39% in the settling time and of 68.95% in the overshoot lower, compared to the PID.

Pan & Xin (2013) developed a nonlinear robust and optimal controller in order to achieve the robust stability and the performance optimality under load uncertainty. The controller was tested through computational simulation with a SCARA-type robot model. For each test the manipulator load was changed, keeping the controller design parameters. The results showed that the proposed controller is able to drive the manipulator to desired position precisely under large load variations, and applicable to a wide range of nonlinear dynamic systems with uncertainties.

A robust adaptive PID control was proposed by Xu & Qiao (2013) in order to solve the problems related to the nonlinearity and the coupling of the manipulator. The proposed controller was able to compensate the unknown bounded disturbances ensuring the global asymptotic stability with respect to the position and velocity and guarantee the robot to track the desired position and velocity trajectories accurately with quite small tracking errors in finite time. When compared with the adaptive PD controllers, the robust adaptive PID controller provided better control performance because the incorporation of an integral action. One disadvantage of this controller is the fact that the derivative gain must be equal to the integral gain ($K_d = K_i$) constraining the flexibility of the controller.

Baghli et al. (2014) presented the concept of MIMO-PID controller: instead of using an independent PID control for each joint (SISO-PID) of the robot, this controller considers the reference position of each joint

and makes the coupling of position errors of each joint. Applying this method, the position errors in the steady-state was reduced, the overshoot was eliminated and the torque applied by the actuator was diminished.

Costa et al. (2018) developed a computed-torque control whose the feedback was composed of and robustified PID controller against uncertain parameters and neglecting dynamics, aiming to reduce the \mathcal{H}_∞ cost. Compared with the conventional PD, the proposed controller presents lower settling time. The tracking error is lower too, but noisier. The controller is less sensible to variations in the parameters of the robot (e.g. payload, inertia moment, friction) than the PD controller. Despite the shortcomings, the proposed control made the actuators use less torque to perform the same movement as the PD control, which results in a reduction in energy consumption.

Other approaches include: Artificial intelligence based position controller (HASAN, 2012; WAI et al., 2010), optimal feedback-linearization control (CHATRAEI; ZÁDA, 2012), computed-torque based controller (CHEN et al., 2014; JAHED et al., 2013), adaptive PID controller (DELAVARI et al., 2012), robust control (FATEH; AZARGOSHASB, 2015; PARK; HAN, 2011) and optimal control (OLIVARES; STAFFETTI, 2015).

3 Linear Quadratic Optimal Control

The Linear Quadratic Regulator (LQR) was introduced by Kalman (1960) in order to provide a solution for a classical problem in control theory: the design of a linear optimal feedback control capable of minimizing the state tracking error of a system with a minimal control effort (KUMAR; JEROME, 2016). This problem was first approached by Wiener and Hall in the 1940s (WIENER, 1949; HALL, 1943), but it was not rigorously formulated from a mathematical point of view.

Studying a plant represented by the following system in state-space form

$$\frac{dx}{dt} = Ax(t) + Bu(t) \quad (1)$$

where $x(t) \in R^m$ is the state vector and $u(t) \in R^n$ is the input control function. The behavior of the model is described by the solution of the Eq. (1), whose general solution has the form (GRANT, 2007):

$$x(t) = \phi(t, t_0)x(t_0) + \int_0^t \phi(t, \tau)B(\tau)u(\tau)d\tau \quad (2)$$

being $\phi(t, \tau)$, defined for all t , the solution of the free system represented by Eq. (1), and

$$\phi(t, t) = I \quad (3)$$

The solution of Eq. (2) is referred to as

$$x(t) = \phi(t; x, t_0) \quad (4)$$

The purpose of the LQR is to produce a control action $u(t)$ for a system with a given non-zero initial state $x(0)$ which back the system to the zero state $x = 0$ in an optimal manner, that is, $u(t)$ must ensure

$$\lim_{t \rightarrow \infty} \phi(t; x, t_0) = 0 \quad (5)$$

It is achieved by minimizing the *quadratic performance index* (LEWIS; M.; ABDALLAH, 1993):

$$J = \frac{1}{2} \int_0^\infty (x^T Q x + u^T R u) dt \quad (6)$$

If $x(t)$ is known and considering the feedback principle (LEWIS; M.; ABDALLAH, 1993), the control law is given as

$$u(t) = -Kx(t) \quad (7)$$

where $K = [K_p \ K_v]$ is the feedback gain matrix.

The determination of K is achieved solving the following equation

$$K = R^{-1} B^T P \quad (8)$$

where P is a symmetric $m \times m$ solution matrix of the *Riccati equation*

$$A^T P + P A - P B R^{-1} B^T P + Q = 0 \quad (9)$$

Q is a symmetric positive semidefinite $m \times m$ matrix ($Q \geq 0$) and R is a symmetric positive definite $n \times n$ matrix ($R > 0$) and they the design parameters of the controller.

4 Linear Quadratic Inverse-Dynamics Control

The dynamics of the manipulator is modeled as

$$M(q)\ddot{q} + V(q, \dot{q}) + G(q) = \tau \quad (10)$$

where $M(q)$ is the inertia matrix $n \times n$ and function of the joint-position, $V(q, \dot{q})$ is the centripetal/Coriolis $n \times 1$ vector and $G(q)$ the gravity vector $n \times 1$ (LEWIS;

M.; ABDALLAH, 1993). The torques necessary to produce the acceleration is given by the $n \times 1$ vector τ (n is the number of joints).

The dynamic equation can be written as

$$M(q)\ddot{q} + N(q, \dot{q}) = \tau \quad (11)$$

where

$$N(q, \dot{q}) = V(q, \dot{q}) + G(q) \quad (12)$$

represents nonlinear terms.

If the kinematic characteristics of the desired motion are given (i.e. accelerations, velocities, positions), and having the dynamic model of the robot, the required torque to produce the motion can be computed through solving the Eq. (10). If the mathematical model of the robot dynamics was well known, the torque could be computed off-line and used to implement a feedforward controller (CRAIG, 2012). This ideal proposition is presented in Figure 1.

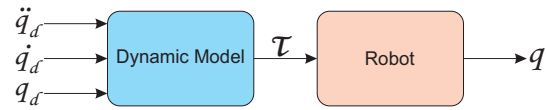


Figure 1: Open loop position control for ideal model

Because some variables are very difficult to be determined (e.g. friction, moment of inertia) and others are neglected during modeling (e.g. backlash), an exact dynamic model is impossible to be achieved. Thus, tracking errors arise, and the model presented in Figure 1 becomes inapplicable, being necessary the development of an outer feedback loop in order to minimize (or ideally, eliminate) the errors.

Taking the trajectory, velocity and acceleration errors, respectively, as

$$e = q - q_d \quad (13)$$

$$\dot{e} = \dot{q} - \dot{q}_d \quad (14)$$

$$\ddot{e} = \ddot{q} - \ddot{q}_d \quad (15)$$

where the index d indicates the desired variable.

Solving Eq. (11) for \ddot{q}

$$\ddot{q} = M^{-1}\tau - M^{-1}N \quad (16)$$

Formulating the *Inverse Dynamics Control Law* as

$$\tau = \hat{M}\ddot{q}_d + \hat{N} + u \quad (17)$$

where \hat{M} and \hat{N} are the estimated inertia and nonlinear terms matrices, and u the feedback control function.

Replacing Eq. (17) in Eq. (16) and making $\hat{M} = M$ and $\hat{N} = N$, the acceleration error becomes

$$\ddot{e} = M^{-1}u \quad (18)$$

The *Linear Quadratic Inverse-Dynamics Control* (LQID) has an inner nonlinear feedforward loop and an outer feedback control loop, as shown in the Figure 2.

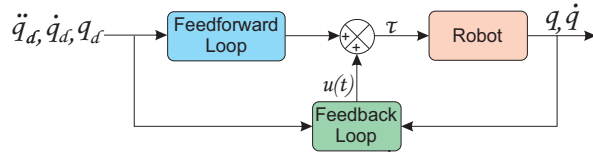


Figure 2: Linear Quadratic Inverse-Dynamics Control. The function $u(t)$ must be chosen in order to stabilize Eq. (14) and Eq. (15) so that Eq. (13) goes to zero.

Writing the errors in state-space form (Eq. (1)), we have:

$$\frac{d}{dt} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} u \quad (19)$$

where

$$u = -Kx = - \begin{bmatrix} Kp & Kv \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \quad (20)$$

The feedback gain K was determined using the *Linear Quadratic design* presented in section 3. The matrices A and B were obtained from Eq. (19), and the matrices Q and R had their values attributed experimentally.

The inertia matrix is time-variable so that, in order to ensure constant feedback gain, it was considered as a constant value matrix, being determined with the robot in the position with maximum moment of inertia:

$$M = \begin{bmatrix} 0.2506 & 0.1224 & 0.0292 \\ 0.1224 & 0.0703 & 0.0195 \\ 0.0292 & 0.0195 & 0.0097 \end{bmatrix}$$

5 Experimental Procedure

To realize the tests 3-link planar robot type UARM-II (Figure 3), from the Robotic Laboratory of the University of São Paulo - São Carlos/Brazil, was used.

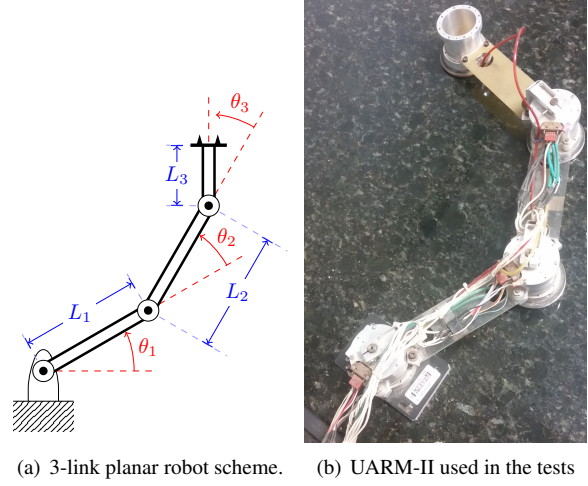


Figure 3: Robot manipulator used in the tests.

Five tests were performed: in the first four tests, was applied a *Linear Quadratic Inverse-Dynamics Control* whose the gains Kp and Kv of the feedback loop were determined using the Linear Quadratic design described above. In each of these tests the values of the Q and R matrices were:

Test 1:

$$Q = \begin{bmatrix} 10I_3 & 0_{3 \times 3} \\ 0_{3 \times 3} & 0.01I_3 \end{bmatrix} \quad R = 0.1I_3$$

$$Kp = 10I_3 \quad Kv = \begin{bmatrix} 2.1078 & 0.8004 & 0.1690 \\ 0.8004 & 0.9121 & 0.1827 \\ 0.1690 & 0.1827 & 0.4817 \end{bmatrix}$$

Test 2:

$$Q = \begin{bmatrix} 10I_3 & 0_{3 \times 3} \\ 0_{3 \times 3} & 0.001I_3 \end{bmatrix} \quad R = 0.1I_3$$

$$Kp = 10I_3 \quad Kv = \begin{bmatrix} 2.0737 & 0.8328 & 0.1675 \\ 0.8328 & 0.8233 & 0.2113 \\ 0.1675 & 0.2113 & 0.3624 \end{bmatrix}$$

Test 3:

$$Q = \begin{bmatrix} 10I_3 & 0_{3 \times 3} \\ 0_{3 \times 3} & 0.0001I_3 \end{bmatrix} \quad R = 0.1I_3$$

$$Kp = 10I_3 \quad Kv = \begin{bmatrix} 2.0700 & 0.8367 & 0.1669 \\ 0.8367 & 0.8126 & 0.2159 \\ 0.1669 & 0.2159 & 0.3472 \end{bmatrix}$$

Test 4:

$$Q = \begin{bmatrix} 10I_3 & 0_{3 \times 3} \\ 0_{3 \times 3} & 0.00001I_3 \end{bmatrix} \quad R = 0.1I_3$$

$$Kp = 10I_3 \quad Kv = \begin{bmatrix} 2.0697 & 0.8371 & 0.1668 \\ 0.8371 & 0.8115 & 0.2164 \\ 0.1668 & 0.2164 & 0.3456 \end{bmatrix}$$

A unique feedback PD control loop was used in the last test. The controller gains were determined by trial and error method and are presented below:

Test 5:

$$Kp = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad Kv = 0.1I_3$$

6 Results and Discussion

As discussed above, two types of controllers were used in the tests: the proposed LQID and the PD feedback (this being the most used in industrial robotics), in order to compare the performance of both. Each subsection deals with a variable of importance.

In relation to the LQID control only the graphs of test 4 are presented here, since for this controller, the variations between the results of the tests performed can be easily understood by analyzing the tables that deal with RMS errors.

6.1 Position

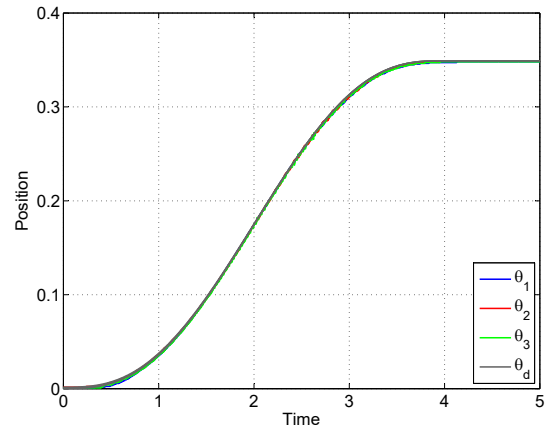
In all tests, the time required to reach steady state was about four seconds. The Linear Quadratic Inverse-Dynamics control (tests 1-4) allowed smaller RMS position errors when compared to the PD feedback control (test 5). Analyzing the RMS error values presented in Table 1, it is possible to verify that, by reducing the weight related to the velocity error, in the Q matrix, the position tracking error is decreased. The test 4 showed the smallest tracking error.

Table 1: RMS position error of joints ($\times 10^{-3}$)

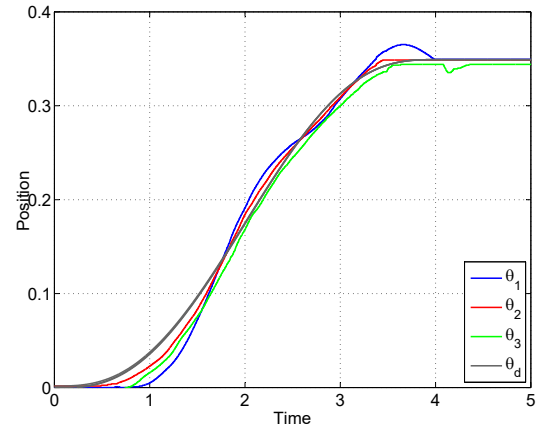
Joint	Test 1	Test 2	Test 3	Test 4	Test 5
q_1	2.2	2.2	2.0	1.5	14.3
q_2	1.5	1.2	1.1	1.2	6.9
q_3	2.1	2.5	2.4	1.7	10.9

Figure 4 shows the position of each of the three joints of the robot relative to the reference θ_d . It is pos-

sible to note that the positions of the joints in test 4 follow very well the desired movement, whereas in test 5 there are reasonable tracking errors during the transient response.



(a) Test 4



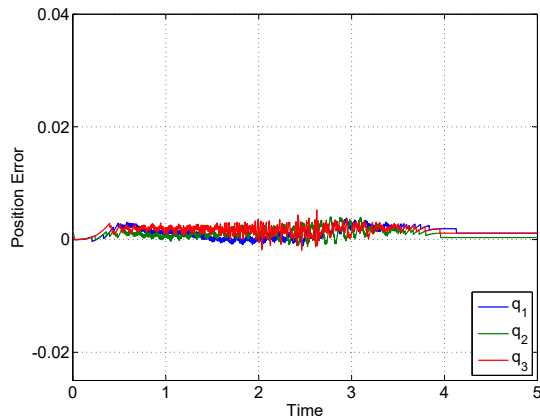
(b) Test 5

Figure 4: Joint positions (rad) x Time (s). $\theta_1, \theta_2, \theta_3$ are the positions of the joints 1, 2 and 3, respectively.

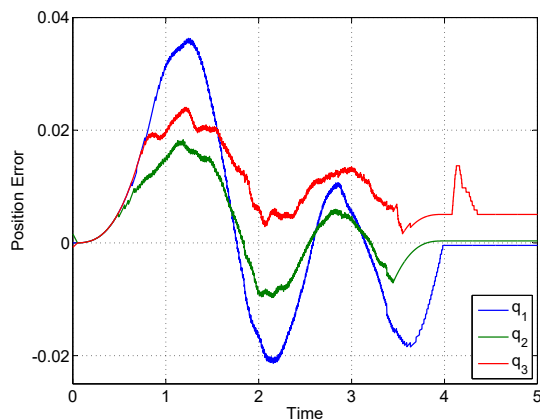
The performance of the position response is analyzed in the Tab. 2 and the tracking position errors are presented in the Fig. 5. It is possible to notice that the proposed controller improved the quality of the transitory and steady-state response, compared to the conventional PID controller: the stationary error and the overshoot were eliminated. These results agree with those achieved by Khairudin, Mohamed & Husain (2011) and confirm the hypothesis 1.

Table 2: Analyze of the angular position response.

Parameter	PID	LQIDC
Steady-state error (rad)	0.006	0
Overshoot (%)	4.34	0
Settling time (s)	4	4



(a) Test 4



(b) Test 5

Figure 5: Position error (rad) x Time (s). q_1 , q_2 , q_3 are the joints 1, 2 and 3, respectively.

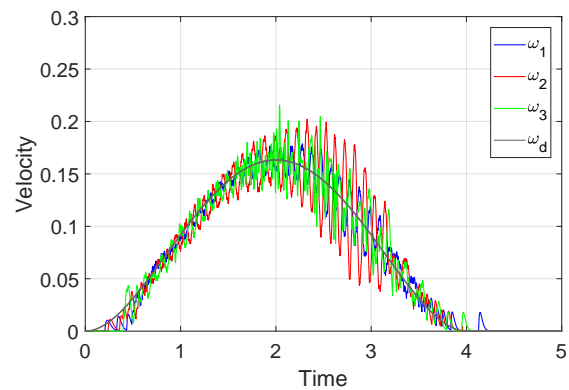
6.2 Velocity

The LQID control allowed smaller velocity errors when compared to the PD feedback control (Table 3). The reduction of velocity weights of the Q matrix provided a better performance until the test 3, from then on, reducing the weights did not cause reduction of the tracking errors, on the contrary, allowed them to increase, as can be observed in the Table 3. The LQID controller despite producing a better result after the reduction of Q

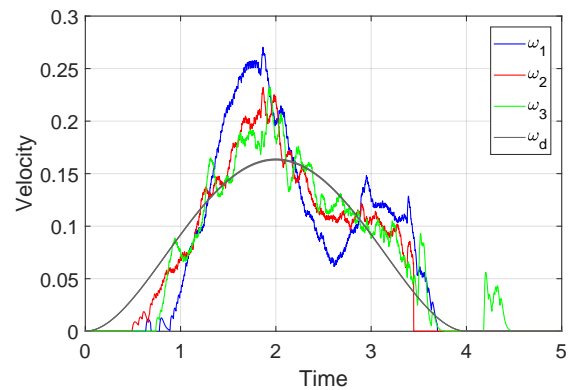
weights, still present increased noise during deceleration. The PD controller with empirical gains presents errors with low noise, but great amplitude (Fig. 6).

Table 3: RMS velocity error of joints ($\times 10^{-3}$)

Joint	Test 1	Test 2	Test 3	Test 4	Test 5
q_1	17.7	10.6	7.0	8.8	42.7
q_2	36.6	20.7	12.8	17.2	21.1
q_3	15.4	12.1	9.7	11.5	23.8



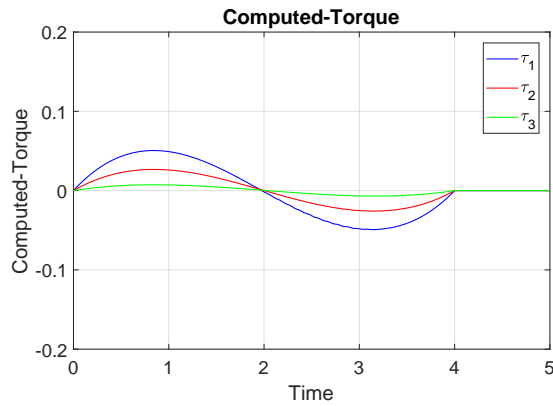
(a) Test 4



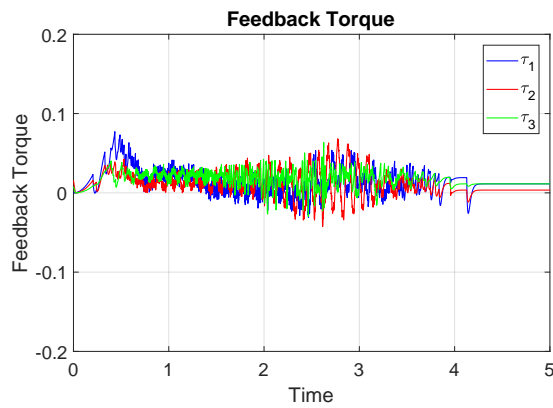
(b) Test 5

Figure 6: Velocities (rad/s) x Time (s). ω_1 , ω_2 , ω_3 are the velocities of the joints 1, 2 and 3, respectively.

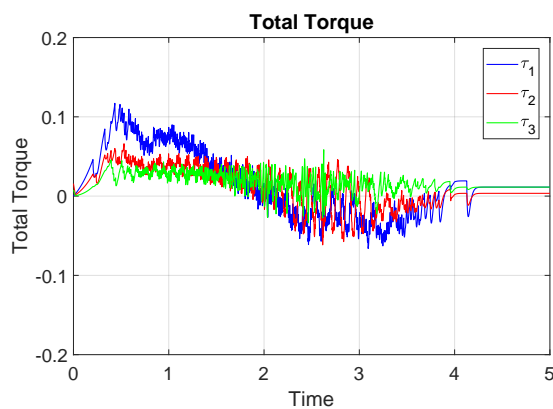
6.3 Torque



(a) Feedforward Torque



(b) Feedback Torque



(c) Total Torque

Figure 7: Torque (N.m) x Time (s) - Test 4. τ_1, τ_2, τ_3 are the torques of the joints 1, 2 and 3, respectively.

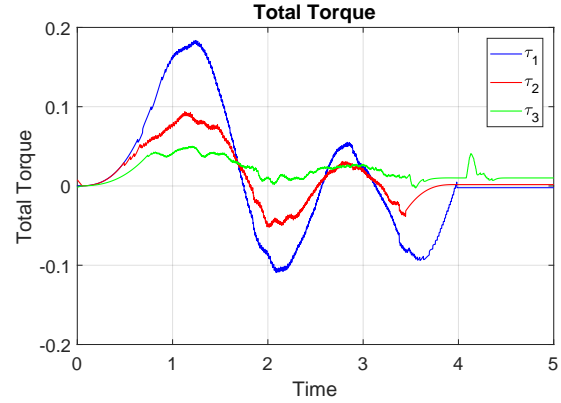


Figure 8: Torque (N.m) x Time(s) - Test 5. τ_1, τ_2, τ_3 are the torques of the joints 1, 2 and 3, respectively.

The Linear Quadratic Inverse-Dynamics controller have two source of torque: the computed coming from the feedforward loop and the feedback that is based on errors. The computed torque remains unchanged between one test an another for the tests 1-4, it is because this control is based on the dynamics of the robot and the desired trajectory, that are the same in all the tests. However, the feedback torque changes from a test to another due to change in gains Kp and Kv .

According to Table 4, it is possible to verify that reducing the velocities weights of the Q matrix the energy consumption to perform the desired motion is reduced, so that in test 4 there was the lower consumption. The PD controller has higher RMS torque values than the LQID one what confirm the hypothesis 2.

If the robotic dynamic model was well known, the feedback torques in the LQID controller would be zero. However, this is unattainable from the practical point of view.

Table 4: RMS total torque of joints ($\times 10^{-3}$)

Joint	Test 1	Test 2	Test 3	Test 4	Test 5
q_1	40.4	43.8	39.7	38.4	72.4
q_2	36.1	25.9	21.9	24.2	34.9
q_3	23.5	26.7	25.6	19.1	22.2

The hypothesis 3 is not confirmed: while the challenge in tuning the conventional PID controller is to select correct feedback gains, in the proposed controller this challenge corresponds to select the weights of the Q and R matrices.

7 Conclusion

This work proposed the development of a controller with two loops, where one is based on the inverse dynamics of the robot and another is based on the LQR. The results proved that this controller has best performance than the conventional feedback PID, with less overshoot and stationary error, as well as low energy consumption.

The disadvantages are presented in the form of selecting the Q and R matrices as well as the need for basic knowledge of robot parameters to determine the mathematical model of its dynamics.

For future work, we indicate the study of ways to select matrices Q and R mathematically, based on the required performance, in order to replace the trial and error method.

References

- BAGHLI, F.; BAKKALI, L. E.; LAKHAL, Y.; NASRI, A.; GASBAOUI, B. Arm manipulator position control based on multi-input multi-output PID strategy. **Journal of Automation, Mobile Robotics & Intelligent Systems**, PIAP - Industrial Research Institute for Automation and Measurements, v. 8, n. 2, p. 36–39, apr 2014.
- CHATRAEI, A.; ZÁDA, V. Global optimal feedback-linearizing control of robot manipulators. **Asian Journal of Control**, Wiley, v. 15, n. 4, p. 1178–1187, nov 2012.
- CHEN, Y.; MA, G.; LIN, S.; NING, S.; GAO, J. Computed-torque plus robust adaptive compensation control for robot manipulator with structured and unstructured uncertainties. **IMA Journal of Mathematical Control and Information**, Oxford University Press (OUP), v. 33, n. 1, p. 37–52, jul 2014.
- COSTA, T. L.; LARA-MOLINA, F. A.; JUNIOR, A. A. C.; TAKETA, E. Robust h_∞ computed torque control for manipulators. **IEEE Latin America Transactions**, Institute of Electrical and Electronics Engineers (IEEE), v. 16, n. 2, p. 398–407, feb 2018.
- CRAIG, J. J. **Introduction to Robotics: Mechanics and Control**. 3. ed. [S.l.]: Pearson Prentice Hall, 2012.
- DELAVARI, H.; GHADERI, R.; RANJBAR, A.; HOSSEINNIA, H.; MOMANI, S. Adaptive fractional pid controller for robot manipulator. In: **The 4th IFAC Workshop Fractional Differentiation and its Applications**. Badajoz, Spain: FDA, 2012.
- FATEH, M.; AZARGOSHASB, S. Discrete time robust control of robot manipulators in the task space using adaptive fuzzy estimator. **Journal of Artificial Intelligence and Data Mining**, International Digital Organization for Scientific Information (IDOSI), v. 3, n. 1, 2015.
- GRANT, C. P. **Theory of Ordinary Differential Equations**. Utah: CreateSpace Independent Publishing Platform, 2007. ISBN 9781502911407.
- HALL, A. C. **The Analysis and Synthesis of Linear Servomechanisms**. Tese (Doutorado), 1943.
- HASAN, A. T. Under-actuated robot manipulator positioning control using artificial neural network inversion technique. **Advances in Artificial Intelligence**, Hindawi Limited, v. 2012, p. 1–6, 2012.
- JAHED, A.; PILTAN, F.; REZAIE, H.; BOROOMAND, B. Design computed torque controller with parallel fuzzy inference system compensator to control of robot manipulator. **International Journal of Information Engineering and Electronic Business**, MECS Publisher, v. 5, n. 3, p. 66–77, sep 2013.
- KALMAN, R. E. Contributions to the theory of optimal control. **Bol. Soc. Mexicana**, p. 102–119, 1960.
- KHAIRUDIN, M.; MOHAMED, Z.; HUSAIN, A. R. Dynamic model and robust control of flexible link robot manipulator. **TELKOMNIKA (Telecommunication Computing Electronics and Control)**, Universitas Ahmad Dahlan, v. 9, n. 2, p. 279, aug 2011.
- KUMAR, E.; JEROME, J. Algebraic riccati equation based q and r matrices selection algorithm for optimal lqr applied to tracking control of 3rd order magnetic levitation system. **Archives of electrical engineering**, v. 65, p. 151–168, 2016.
- LAMMERTS, I. **Adaptive computed reference computed torque control of flexible manipulators**. Tese (Doutorado), 1993.
- LEWIS, F.; M., D. D.; ABDALLAH, C. **Robot Manipulator Control Theory and Practice**. New York: Prentice-Hall, 1993.
- MOOLAM, R. K. **Dynamic Modeling and Control of Flexible Manipulators**. Tese (Doutorado), 2013.

OLIVARES, A.; STAFFETTI, E. Embedded optimal control of robot manipulators with passive joints. **Mathematical Problems in Engineering**, Hindawi Limited, v. 2015, p. 1–21, 2015.

PAN, H.; XIN, M. Nonlinear robust and optimal control of robot manipulators. **Nonlinear Dynamics**, Springer Nature, v. 76, n. 1, p. 237–254, nov 2013.

PARK, S.; HAN, S. Robust-tracking control for robot manipulator with deadzone and friction using backstepping and RFNN controller. **IET Control Theory & Applications**, Institution of Engineering and Technology (IET), v. 5, n. 12, p. 1397–1417, aug 2011.

WAI, R.-J.; HUANG, Y.-C.; SHIH, C.-Y.; YANG, Z.-W. Adaptive fuzzy-neural-network velocity sensorless control for robot manipulator position tracking. **IET Control Theory & Applications**, Institution of Engineering and Technology (IET), v. 4, n. 6, p. 1079–1093, jun 2010.

WIENER, N. **Extrapolation, Interpolation and Smoothing of Stationary Time Series**. Massachusetts: MIT Press, 1949.

XU, J.; QIAO, L. Robust adaptive PID control of robot manipulator with bounded disturbances. **Mathematical Problems in Engineering**, Hindawi Limited, v. 2013, p. 1–13, 2013.